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Strength Predictions of Composite Laminates with Unloaded Fastener Holes

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Results of a theoretical and experimental program to develop static strength methodology for composite laminates with unloaded fastener holes are reported. A closed-form analysis method is used to predict laminate stress and strain distributions around an unloaded hole in an infinite anisotropic plate. Analysis is performed and failure predicted on a ply-by-ply basis to determine gross laminate strength. This methodology has been computerized with the capability to handle material strength and stiffness anisotropy, general in-plane loadings (tension, compression, biaxiality, shear), and arbitrary hole sizes. Strength predictions are made for tensile-loaded AS/3501-6 graphite-epoxy laminates with holes. The effect of varying layup, hole size, and loading direction on failure load and initiation point is evaluated. Predictions are in excellent agreement with test data.

Nomenclature

a	= hole radius
E_1^t	= tensile modulus of elasticity in fiber direction
E_1^c	= compressive modulus of elasticity in fiber direction
E_2^t	= tensile modulus of elasticity transverse to fibers
E_2^c	= compressive modulus of elasticity transverse to fibers
F	= Airy's stress function
F_1^{tu}	= allowable tensile strength in fiber direction
F_1^{cu}	= allowable compressive strength in fiber direction
F_2^{cu}	= allowable compressible strength transverse to fibers
F_2^{tu}	= allowable tensile strength transverse to fibers
F_{12}^{su}	= allowable shear strength
G_{12}	= shear modulus
i	= imaginary unit
P	= applied uniaxial inplane stress field
R_c	= characteristic dimension
R_1, R_2	= complex roots of characteristic equation
S_{jk}	= compliance coefficients
X, Y	= laminate Cartesian coordinate axes
α	= angular orientation of stress field with X-axis
ν_{12}	= Poisson's ratio
$\sigma_X, \sigma_Y, \tau_{XY}$	= laminate stress components

Background

COMPOSITES differ from metals because of their anisotropic mechanical properties and their general lack of ductility. This latter characteristic limits redistribution of locally high stresses in areas of stress concentration such as around a fastener hole. Local stress risers do not reduce significantly the static strength of metal joints because metals tend to yield. Composites, however, are very sensitive to stress risers, which are a dominant factor limiting static strength of composite bolted joints.

Theoretical methods[1] currently used to determine stress distributions about a fastener hole in a composite laminate include both closed-form solutions and numerical procedures. Analytic closed-form procedures are generally preferred because they are most useful for parametric studies and are continuous between specified boundary conditions. After detailed stress distributions are determined, strength is assessed using material failure criteria.

Strength predictions based upon elastic analysis at points of peak anisotropic stress concentration have been shown to be highly conservative.[2] Theoretical elastic stress concentrations are approximately 25-50% higher than test data would indicate. The higher-than-predicted strength has been attributed to some limited nonlinear composite shear behavior,[3] and to localized and progressive ply damage.[4] To date, most analytic strength predictions have failed to account for this nonlinear or inelastic material behavior at the hole boundary, or have been limited to uniaxial loadings. Additionally, current methods require laminate strength data for each layup tested.

In this paper, strength analysis will be presented which is performed on a ply-by-ply basis. Material behavior in the region immediately surrounding the fastener hole is accounted for by using a failure hypothesis based upon the "characteristic dimension" concept of Wu[5] and Whitney and Nuismer.[6] Strength predictions are made for tensile loaded AS/3501-6 graphite-epoxy laminates with an unloaded and unfilled fastener hole. Only mechanical property data for the basic lamina (unidirectional ply) of a material system and for one laminate configuration are required to use the methodology to predict strength for any other laminate variation. Variables examined include layup, hole size, and loadings off the principal material axes. For all cases, predictions are correlated with test data.

Method of Analysis

Static strength of a laminate with an open fastener hole in an anisotropic plate is predicted using a closed-form analytic approach. The method of analysis is based on 1) anisotropic theory of elasticity, 2) lamination plate theory, and 3) a failure hypothesis.

The elastic solution for the stress field in a homogeneous, anisotropic infinite plate has been solved using two-dimensional anisotropic theory of elasticity.[7] This analysis,

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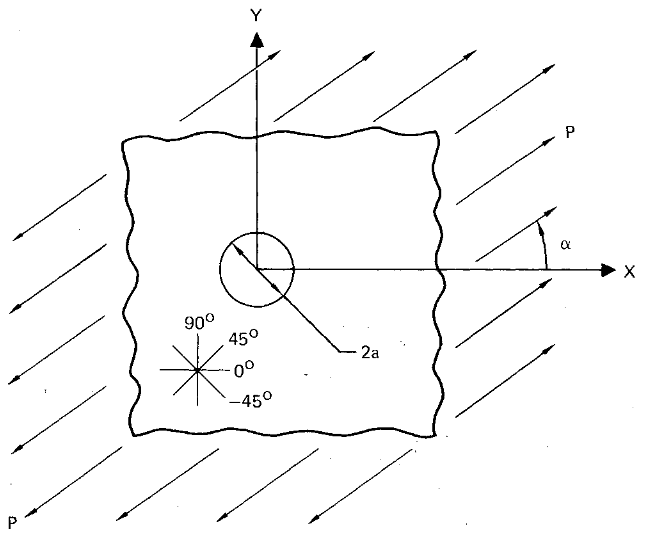

Fig. 1 Uniaxially loaded infinite plate.

using plane stress assumptions, applies to a plate subjected to a remote uniaxial in-plane stress field P at an arbitrary orientation α with the X axis (Fig. 1). This closed-form solution satisfies equilibrium and compatibility requirements by deriving a stress function F, which satisfies the generalized biharmonic equation for anisotropic materials,

$$S_{22}\frac{\partial^4 F}{\partial X^4} - 2S_{26}\frac{\partial^4 F}{\partial X^3 \partial Y} + (2S_{12}+S_{66})\frac{\partial^4 F}{\partial X^2 \partial Y^2}$$

$$-2S_{16}\frac{\partial^4 F}{\partial X \partial Y^3} + S_{11}\frac{\partial^4 F}{\partial Y^4} = 0$$

where S_{jk} are laminate compliance coefficients. The general expression for the function F depends upon the roots of the associated characteristic equation. For this case, the stress function equals

$$F = 2Re\{F_1(Z_1) + F_2(Z_2)\}$$

where $F_1(Z_1)$, $F_2(Z_2)$ are analytic functions of the complex variables $Z_1 = X + R_1 Y$ and $Z_2 = X + R_2 Y$, respectively, with R_1 and R_2 being complex roots obtained from the characteristic equation. Introducing the functions

$$\phi_1(Z_1) = \frac{\partial F(Z_1)}{\partial Z_1} \qquad \phi_2(Z_2) = \frac{\partial F(Z_2)}{\partial Z_2}$$

the general expressions for the stress components are obtained:

$$\sigma_X = 2Re\{R_1^2\phi_1'(Z_1) + R_2^2\phi_2'(Z_2)\} + P\cos^2\alpha$$

$$\sigma_Y = 2Re\{\phi_1'(Z_1) + \phi_2'(Z_2)\} + P\sin^2\alpha$$

$$\tau_{XY} = -2Re\{R_1\phi_1'(Z_1) + R_2\phi_2'(Z_2)\} + P\ \sin\alpha\ \cos\alpha$$

Superscript primes represent derivatives with respect to the complex arguments. Savin[7] derived the functions $\phi_1'(Z_1)$ and $\phi_2'(Z_2)$ as:

$$\phi_1'(Z_1) = \frac{-iP}{2(R_1-R_2)(1+iR_1)}\left\{1 - \frac{Z_1}{\sqrt{Z_1^2 - a^2(1+R_1^2)}}\right\}$$

$$\phi_2'(Z_2) = \frac{iP}{2(R_1-R_2)(1+iR_2)}\left\{1 - \frac{Z_2}{\sqrt{Z_2^2 - a^2(1+R_2^2)}}\right\}$$

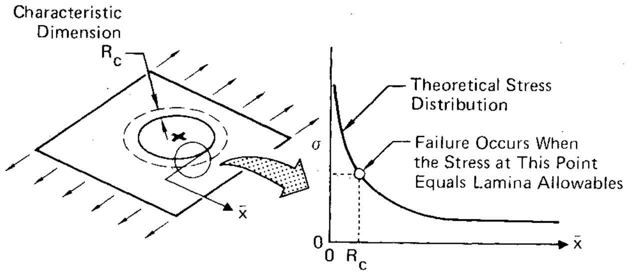

Fig. 2 Failure hypothesis.

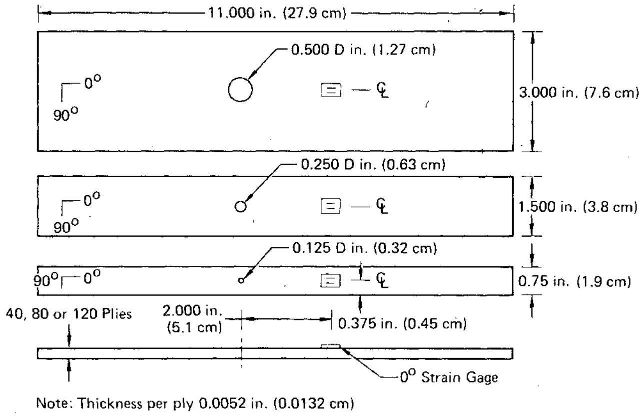

Fig. 3 Test specimen geometry.

This solution is valid only for homogeneous media, but is assumed valid also for midplane symmetric laminates. These equations give the complete elastic stress distribution in an uniaxially loaded infinite, two-dimensional, anisotropic, laminate with a circular hole. Laminate strains are calculated using material compliance constitutive relations. Laminate compliance coefficients S_{jk} are derived using classical lamination plate theory[8] with unidirectional material elastic constants, ply angular orientations, and ply thicknesses. Assuming that laminate strain remains constant through the thickness, strains for individual plies along lamina principal material axes are calculated using coordinate transformations. Stress distributions resulting from an arbitrary set of biaxial in-plane loads are easily obtained using the principle of superposition.

To account for composite material inelastic or nonlinear behavior at the hole boundary, the "characteristic dimension" hypothesis by Whitney and Nuismer[6] has been adapted. Their hypothesis states that failure of a composite material with a stress concentration can be correlated with analytical predictions of point stresses at a characteristic dimension from the edge of a stress concentration (Fig. 2). This paper extends the application of the characteristic dimension failure hypothesis to permit strength predictions for anisotropic laminates under general biaxial loads, without requiring extensive laminate test data. To do this, laminate failure is predicted by comparing elastic stress distributions with material failure criteria on a ply-by-ply basis at a characteristic dimension away from the hole boundary.

Location of failure initiation was predicted by evaluating all stress components at each point around the perimeter of the hole. In metals, assumptions on failure location can be made because peak stresses occur on the hold boundary, at 90° to applied load direction, Similar assumptions cannot be made for composite materials because of material anisotropic strength and stiffness properties. Failure initiation does not always result from a single peak stress component, nor on the hole boundary in the same place as metals. This is particularly true for biaxially loaded laminates or for uniaxial loads not aligned with principal material axes.

Table 1 Test matrix for unloaded hole investigations

Laminate Group	% Layers of 0°	±45°	90°	No. Plies	Hole Dia. ~ in. (cm) 0.0	0.125 (0.32)	0.250 (0.63)	0.500 (1.27)	Loading Ten.	Com.	No. of Tests
A	70	20	10	40	√	√	√	√	√	√	24
B	50	40	10	40	√	√	√	√	√	√	24
	↓	↓	↓	80		√	√	√	√	√	18
				120		√	√	√	√	√	18
C	30	60	10	40	√	√	√	√	√	√	24
											108
D	50	10	40	40	√		√		√	√	12
E	40	20	40	40	√		√		√	√	12
F	40	50	10	40	√		√		√	√	12
G	30	30	40	40	√		√		√	√	12
H	20	40	40	40	√		√		√	√	12
I	20	70	10	40	√		√		√	√	12
J	10	80	10	40	√		√		√	√	12
											84
									Total No. Tests		192

Each Test Condition Contains a Replication of 3 Specimens

Table 2 Graphite-epoxy lamina mechanical properties

Lamina Property	Symbol	Value	
Elastic Constants	E_1^t	18.69 x 10^6 psi	12.89 x 10^4 MPa
	E_1^c	17.57 x 10^6 psi	12.11 x 10^4 MPa
	E_2^t	1.90 x 10^6 psi	1.31 x 10^4 MPa
	E_2^c	1.90 x 10^6 psi	1.31 x 10^4 MPa
	G_{12}	0.85 x 10^6 psi	0.59 x 10^4 MPa
	ν_{12}	0.3	0.3
Strengths	F_1^{tu}	232.5 ksi	1603 MPa
	F_1^{cu}	291.7 ksi	2011 MPa
	F_2^{tu}	9.5 ksi	66 MPa
	F_2^{cu}	38.9 ksi	268 MPa
	F_{12}^{su}	17.3 ksi	119 MPa

The Tsai-Hill material failure criterion[9] was used to obtain all strength predictions reported in this paper. Tension or compression stress allowables used in this criterion were selected depending upon the sign of individual stress field components being evaluated. In all cases, laminate failure was assumed to occur when first ply fiber failure was predicted.

This "Bolted Joint Stress Field Model" (BJSFM) has been incorporated into a computer program which has the capability to handle material strength and stiffness anisotropy, general in-plane loadings (tension, compression, biaxiality, shear), multimaterial (hybrid) laminates, and arbitrary hole sizes. Only mechanical properties for the basic lamina (unidirectional ply) properties and for one laminate configuration are required to obtain strength predictions of arbitrary laminates with open fastener holes.

Test Results and Analytical Correlation

An experimental program was conducted to verify accuracy of analytical strength predictions. Graphite-epoxy AS/3501-6 was chosen as the test material. The test plan was defined to provide a broad base for verification. Eleven different layups in the 0°, ±45°, 90° family, ranging from 10% 0° plies to 70% 0° plies, were tested. Ten layups (Table 1) were used to investigate effects of layup, hole size, and thickness on unloaded hole laminate strength and were tested under both tension and compression loadings. An eleventh layup was tested under uniaxial tension loading at various angles off the principal material axis (axis parallel to 0° plies) to evaluate the program's capability to predict effects of shear-extensional coupling on laminate strength. Specimens were configured with a constant width-to-diameter ratio of six and all specimens were instrumented to provide modulus data. Specimen dimensions and strain gage locations are shown in Fig. 3. In this paper, only data from tensile loaded tests will be reported.

Strength predictions were made for all layup variations and test conditions. Laminate elastic properties were calculated using lamination plate theory and unidirectional ply lamina elastic constants. Laminate failure was based on the Tsai-Hill material failure criterion, applied at a characteristic dimension away from the hole, using unidirectional lamina strength data. All lamina data used in this analysis are shown in Table 2. These data are average values derived from tests performed on unnotched unidirectional laminate sandwich beams, and 0°-90° rail-shear specimens.

Correlation of predicted strength of all layups is shown in Fig. 4. Correlation of predicted effects of hole size on laminate strength is shown in Fig. 5. Data from any one laminate in conjunction with lamina mechanical properties is sufficient to permit strength predictions using the BJSFM procedure for any other layup. For the AS/3501-6 material system, a characteristic dimension of 0.05 cm (0.02 in.) was

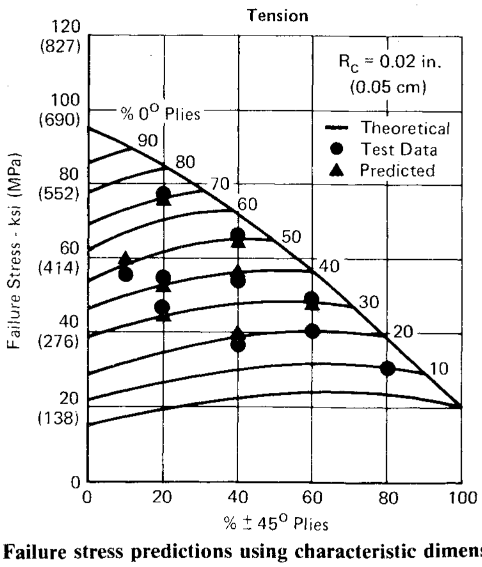

Fig. 4 Failure stress predictions using characteristic dimension (R_c) are verified by tests.

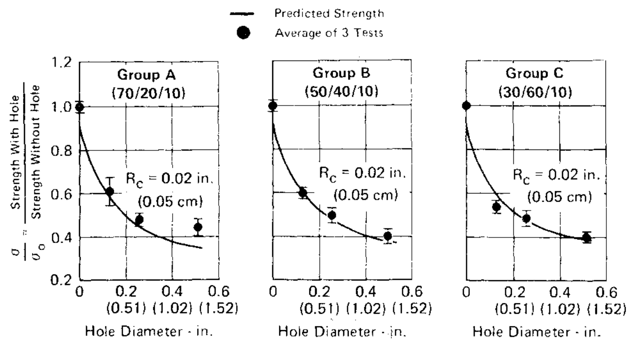

Fig. 5 Correlation of predicted effect of hole sizes and lay-up on laminate tension strength.

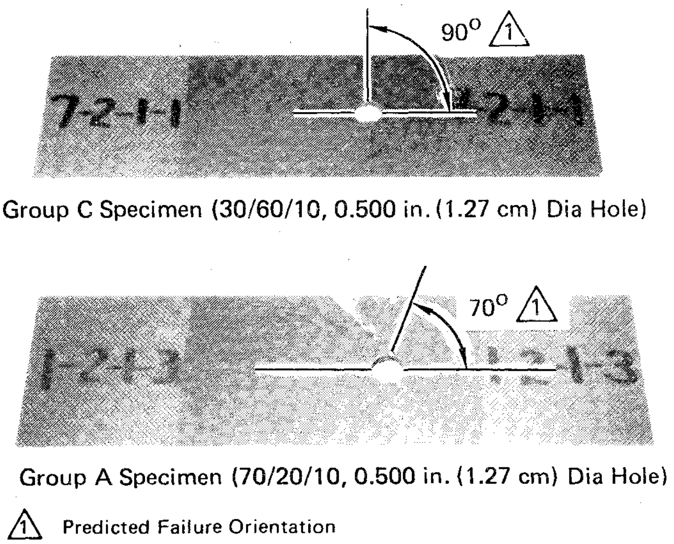

Fig. 6 Test specimen failure locations.

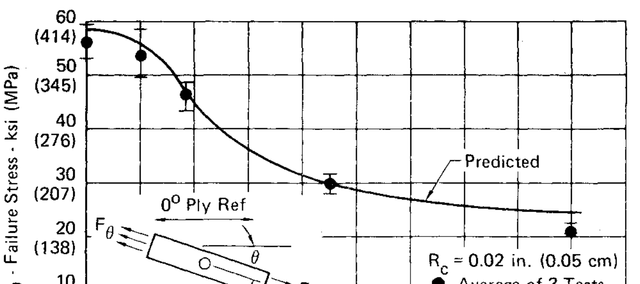

Fig. 7 Laminate strength predictions for off-axis loading.

used for all failure analysis. This value was obtained empirically by using hole size effects test data obtained from the 50/40/10 laminate.

Location of failure initiation depends on layup and hole size. Predicted failure location occurred on the hole boundary and at 90° to the applied load direction for all but two cases. The exceptions are the 70/20/10 layup with 0.64 or 1.27 cm (0.250 or 0.500 in.) diameter holes. Predicted origins of failure were at 65° for the 0.64 cm (0.250 in.) diameter hole and at 70° for the 1.27 cm (0.500 in.) diameter hole. In these two cases, failure was predicted in the ±45° plies. All predicted locations of failure correlated with test data (e.g., Fig. 6).

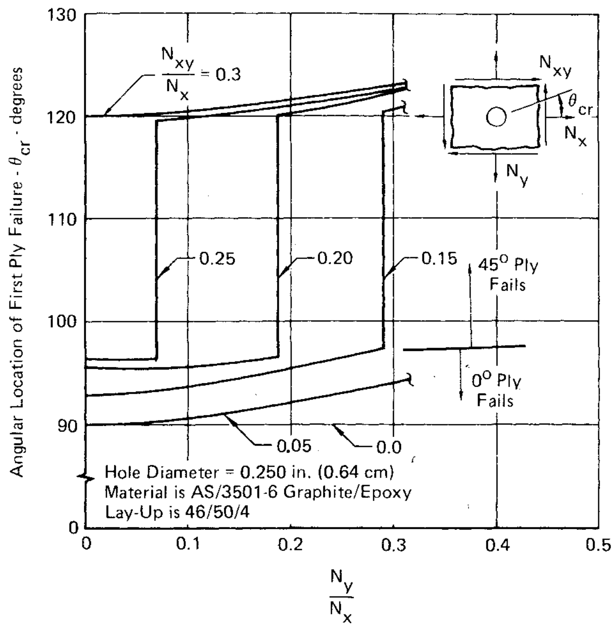

Fig. 8 Failure location depends on biaxial stress state.

The ability of the BJSFM procedure to make strength predictions for a biaxially loaded laminate with an open fastener hole was also verified by test. This stress state was achieved with uniaxial loading acting at various angles relative to the laminate principal material axis. Due to orthotropic material behavior, off-axis uniaxial loading causes shear-extensional coupling which produces a biaxial stress state in the test specimen gage area. Using the 0.05 cm (0.02 in.) characteristic dimension, correlation of test data with theory as shown in Fig. 7 is excellent.

To understand the complex nature of first ply failure location and to demonstrate utility of the BJSFM procedure, a parametric analysis was performed on a cross-plied laminate. The predicted locations of first ply failure at the fastener hole boundary are shown in Fig. 8. At certain biaxial load ratios, a wide arc of the hole boundary is predicted to be critical and as biaxial load ratios increase, failure initiation point and critical plies change. Design charts such as Fig. 8 can aid in the selection of laminates which exhibit desired failure modes for particular loading conditions.

Conclusions

A closed-form strength analysis procedure for laminates with unloaded fastener holes has been developed and verified by a coordinated experimental program. This procedure predicts laminate strength, location of points of failure initiation, and critical plies. It provides failure mode information and stress and strain distributions for arbitrary composite laminates with unloaded and unfilled fastener holes. Strength analysis is performed on a ply-by-ply basis using the Tsai-Hill failure criterion. An experimental program was used to correlate strength predictions with a wide variety of layups from the 0°, ±45°, 90° family of ply orientations fabricated from AS/3501-6 graphite-epoxy. In all cases, correlation of theory with test data was excellent and verified that:

1) Mechanical properties for the unidirectional lamina and only one laminate configuration are needed to permit strength predictions of arbitrary laminates within the same material system.

2) A failure hypothesis based on a constant characteristic dimension of 0.05 cm (0.02 in.) can be used to predict tensile strength of arbitrary layups with an unloaded hole.

3) Location of failure initiation is dependent upon layup, loading, and geometry, therefore strength analysis of com-

plete stress fields must be performed to determine the most critical combination of stresses.

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